Name:

Fall 2016 Math 245 Exam 2

Please read the following directions:
Please write legibly, with plenty of white space. Please print your name on the designated line, similarly to your quizzes. Please fit your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. The use of notes, calculators, or other materials on this exam is strictly prohibited. This exam will last at most 50 minutes; pace yourself accordingly. Please leave **only** at one of the designated times: 1:20pm, 1:40pm, or 1:50pm. At all other times please stay in your seat (emergencies excepted), to ensure a quiet test environment for others. Good luck!

Problem	Min Score	Your Score	Max Score
1.	5		10
2.	5		10
3.	5		10
4.	5		10
5.	5		10
6.	5		10
7.	5		10
8.	5		10
9.	5		10
10.	5		10
Exam Total:	50		100
Quiz Ave:	50		100
Overall:	50		100

Problem 1. Carefully define the following terms:

a. Proof by Contradiction theorem

b. Uniqueness Proof theorem

c. proof by strong induction

d. Set S is *well-ordered* by $<$.

Problem 2. Carefully define the following terms:

a. recurrence

b. $a_n = \Theta(b_n)$

c. $S = T$, for sets S, T

d. $S \cup T$, for sets S, T

Problem 3. Let $n \in \mathbb{Z}$. Prove that $\frac{(n+1)(n-2)}{2} \in \mathbb{Z}$.

Problem 4. Use mathematical induction to prove that $\forall n \in \mathbb{Z}$ with $n \geq 3$, $2^n > 5$.

Problem 5. Suppose that an algorithm has runtime specified by the recurrence relation $T_n = n^{1/2} T_{n/2} + 2$. Determine what, if anything, the Master Theorem tells us.

Problem 6. Let S, T be sets with $S \cap T = S$. Prove that $S \subseteq T$.

Problem 7. Let S be a set. Prove that $S \setminus \emptyset = S$.

Problem 8. Let $x \in \mathbb{R}$. Prove that $2\lfloor x \rfloor \leq \lfloor 2x \rfloor \leq 2\lfloor x \rfloor + 1$.

Problem 9. Let $x \in \mathbb{R}$ with $x > -1$. Prove that $\forall n \in \mathbb{N}_0, (1 + x)^n \geq 1 + nx$.

Problem 10. Prove that $3^n \neq O(2^n)$.